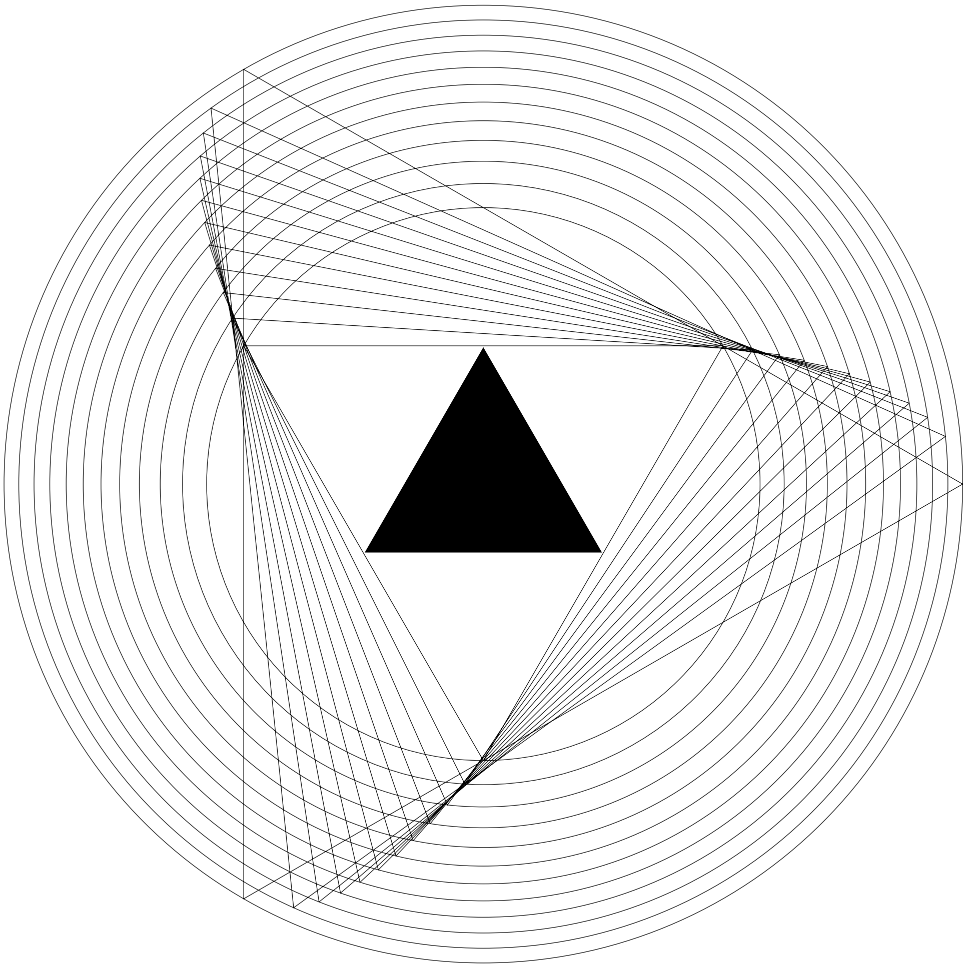

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This bulletin is an edited excerpt from *99 Variations on a Proof*, to be published by Princeton University Press in February 2019. The book explores the possible meanings of style for mathematics — as a science and an art — by approaching the solution of a single equation in nearly a hundred different ways.

Cover: Mystical, one of 99 variations in the upcoming book here with notes omitted (and faith required).

Preface

On April 19, 1610, upon receiving an advance copy of Galileo's *Starry Messenger*, Johannes Kepler composed a fan letter. "I may perhaps seem rash in accepting your claims so readily with no support of my own experience," Kepler wrote to Galileo. "But why should I not believe a most learned mathematician, whose very style attests the soundness of his judgement?"[1] We are not accustomed today to thinking of a mathematician's work in terms of style. A proof is a form of argument, but the truth of the theorem it proves would hardly seem to depend on any of rhetorical, let alone stylistic, features of its proof. The received wisdom is that mathematics, the universal language of science, has one style—*the mathematical style*—characterized by symbolic notation, abstraction, and logical rigor.[2]

This book aims to challenge that conception of mathematics. While a belief in the universality and unity of *ars mathematica* is not without reason, a moment's reflection gives rise to some basic questions. Where did "the" mathematical style come from? How has it developed with the growth of mathematical knowledge? What opportunities does it open or foreclose? How has its potential evolved with changes in the forms of writing, and, therefore, ways of reading, mathematics? What are its expressive, cognitive, and imaginative possibilities?

These questions, at heart, concern the *literature* of mathematics. To survey this literature—a vast body of material ranging in subject from algebra to geometry, number theory to physics, logic to statistics, and dating from Babylonian tablets of the Bronze Age to the peer-reviewed journals and electronic preprints of today—is clearly beyond the scope of a book this size. Instead, I will describe a cross section of mathematics using a method inspired by Raymond Queneau's*Exercises in Style.* This literary work from 1947 takes the same simple story—that of a peculiar individual who is first seen in a dispute on a bus, and then later in conversation with a friend about the position of a coat button—and transforms it in ninety-nine different ways. Queneau's stylistic exercises exemplify various forms of prose, poetry, and speech, as well as more striking contortions, such as "Onomatopoeia," "Dog Latin," and "Permutations by Groups of 2, 3, 4 and 5 Letters." Queneau, in addition to being an author and poet, was also an amateur mathematician, and together with the mathematical historian Franois Le Lionnais he cofounded the experimental writing group known as the Oulipo. The name of the mostly French writing group is an acronym for *Ouvroir de Litérature Potentielle* (Workshop for Potential Literature), and its membership includes writers, artists, and mathematicians such as Georges Perec, Italo Calvino, Marcel Duchamp, Jacques Roubaud, Claude Berge, and Michle Audin. The stated purpose of the group is to explore the possibilities for literature derived from mathematically inspired rules or constraints.[3] As soon as I learned about the Oulipo and Queneau's book, I wanted to see what effect constrained writing strategies would have on a mathematical narrative—a proof.

The theme I chose for *99 Variations on a Proof* is an algebraic equation known as a cubic equation, and every chapter proves the same minor—some might say trivial—theorem about its solutions. Many proofs, from Ancient to Modern, emerge from the mathematical literature on cubics. In some cases this happened quite directly, the extreme example being Found, which I discovered, ready-made, on a page of the most famous Renaissance treatise on algebra. More often than not, however, variations required considerable interpretation and invention. Sometimes this was because the style originated in a subject area peripheral to cubics, as in Axiomatic or the physics-based proof Electrostatic. Still more distant translations were needed to convey styles outside of mathematics entirely, such as the musical score Auditory and the architectural Axonometric.

Some proofs aim to satisfy a particular standard of rigor, some fall short of today's standard, and some have other aims entirely.

Each variation, with relatively few exceptions, appears on a single page with a brief discussion on the reverse side of the page. The secondary text includes explanatory details, source information, and my comments on the nature and significance of each style. Cross references to related variations invite readers to deviate from the idiosyncratic order that I've given to the chapters and find their own paths through the book.

This is not a mathematical treatise on cubic equations, and my choice of the particular cubic here was made almost arbitrarily. Despite the historical threads implied by the chapter titles above, this is not a book of mathematical history; while the ontological status of content and style is a matter of some debate, this is also not a work of philosophy. It is a book *about* mathematics, its attitudes, norms, perspectives, and practices—in short, its culture.[4]

Other comparative studies of mathematical proof have addressed the relationship between content and form in different ways. In 1938, one H. Pétard published "A Contribution to the Mathematical Theory of Big Game Hunting," which offers thirty-eight applications of modern mathematics and physics to the problem of catching a lion.[5] During the writing of *99 Variations on a Proof*, two other mathematical renderings of Queneau's *Exercises* appeared: *Rationnel mon Q* by Ludmila Duchne and Agnès Leblanc and *Exercises in (Mathematical) Style* by John McCleary. While there is necessarily some overlap between these books, it is surprising that studies of style could themselves vary in style so much. This in and of itself further confirms the potential of the basic premise of Queneau's original.

What distinguished the style of that most learned mathematician, Galileo? "For him, good thinking means quickness, agility in reasoning, economy in argument, but also the use of imaginative examples,"[6] according to Italo Calvino. This Oulipian finds the clearest statement of Galilean style in the following passage of *The Assayer* from 1623: while criticizing an adversary's reliance on authority to carry an argument, Galileo asserts, "but discoursing is like coursing, not like carrying, and one Barbary courser can go faster than a hundred Frieslands." Calvino calls this Galileo's "declaration of faith—style as a method of thought and as literary taste."[7] This is a faith that I have tried to keep.

My motivation for this project, from beginning to end, has been to try to conceptualize mathematics as a literary or aesthetic medium. There is no shortage of evidence that professional mathematicians describe their work in aesthetic terms, but the terms they use, at least publicly, are very limited. The oft-repeated "beauty" and "elegance," may be important components of mathematical taste, but they fail to convey its range or subtlety or how it relates to literary and aesthetic experiences beyond mathematics.[8] The ninety-nine (or, if you admit an OMITTED proof the same status as the others, one hundred) proofs serve to highlight the material differences in logic, diction, imagery, and even typesetting, that give tone and flavor to mathematics.[9] I hope that readers with little or no predisposition to the subject matter will begin to perceive these stylistic differences by merely paging through examples, stopping to look more closely at proofs that reflect—or offend—their sensibility and moving lightheartedly forward from any that do not. The reader inclined to delve deeper might recognize that the book itself is a mathematical game. In any case, if mathematics is made more vivid as a result of it passing through the reader's hands, the book will have served its intended purpose.

Notes

1. Galileo Galilei, *Sidereus Nuncius, or The Sidereal Messenger* trans. Albert Van Helden (Chicago: University of Chicago Press, 1989), 98, quoted in Dava Sobel, *Galileos Daughter: A Historical Memoir of Science, Faith and Love* (New York: Bloomsbury Publishing USA, 1999).
2. Here I am paraphrasing the twentieth century mathematician and philosopher Gian-Carlo Rota's *Indiscrete Thoughts* (Boston: Birkhäuser Boston, 2008).
3. Franqis Le Lionnais, "Lipo: First Manifesto," in *Oulipo: A Primer of Potential Literature*, ed. Warren Motte (Champaign, IL: Dalkey Archive Press, 1986), 26–28.
4. Here I am paraphrasing the preface of Philip Davis and Reuben Hersh's wonderful 1981 book *The Mathematical Experience* (Boston: Birkhäuser, 1981), which has served me as both a style guide to writing about mathematics and a source of encouragement throughout this project.
5. H. Pétard, "A Contribution to the Mathematical Theory of Big Game Hunting," *American Mathematical Monthly* 45, no. 7 (August 1938): 446–447.
6. Italo Calvino, *Six Memos for the Next Millennium*, trans. Geoffrey Brock (Boston: Mariner Books, 1988), 52.
7. Ibid., 51.
8. Michael Harris has noted "Aesthetic judgement in mathematics is hampered by its meager lexicon; it doesn't inspire 'lofty' habits in the use of language" and "we have made no progress at all toward elucidating what beauty in mathematics has to do with beauty elsewhere," *Mathematics without Apologies: Portrait of a Problematic Vocation* (Princeton, NJ: Princeton University Press, 2015), 307.
9. The American mathematician William Thurston discusses the importance of "tone and flavor" in his 1994 article "On Proof and Progress in Mathematics," *Bull. Amer. Math. Soc.* 30 (1994): 161–177.
10. Siobhan Roberts, *Genius at Play: The Curious Mind of John Horton Conway* (New York: Bloomsbury Publishing USA, 2015), 286.
11. Raymond Queneau, "The Place of Mathematics in the Classification of the Sciences," in *Letters, Numbers, Forms: Essays 1928–1970* trans. Jordan Stump (Urbana, IL: University of Illinois Press, 2007), 99. This paper appears alongside the Bourbaki manifesto, "The Architecture of Mathematics," in *Great Currents of Mathematical Thought*, edited by François Le Lionnais.
12. Oliver Byrne, *The First Six Books of the Elements of Euclid: In Which Coloured Diagrams and Symbols Are Used Instead of Letters for the Greater Ease of Learners* (London: William Pick- ering, 1847), vii.
13. Richard Feynman, *What Do You Care What Other People Think?* (New York: Norton, 1988), 59.
14. Gerhard Gentzen, "Untersuchungen über das logische Schlieen (Investigations into Logical Inference)" *Mathematische zeitschrift* 39, no. 1 (1935): 176–210.
15. Sobel, 39 and Kevin S. Brown, "Galileos Anagrams and the Moons of Mars," http://www.mathpages.com/home/ kmath151/kmath151.htm.
16. Christiaan Huygens, "De Saturni Luna Observatio Nova," in *Oeuvres Completes de Christiaan Huygens* 15 (1656): 172–7.
17. John Fauvel and Jeremy Gray, *The History of Mathematics: a Reader* (Basingstoke, UK: Palgrave Macmillan Education, 1987), 407.
18. Archimedes, *The Works of Archimedes: Volume 2, On Spirals*, trans. Reviel Netz (Cambridge University Press, 2017), 28.

19. Ralph Abraham, “Mathematics and the Psychedelic Revolution: Recollections of the Impact of the Psychedelic Revolution on the History of Mathematics and my Personal Story,” *MAPS Bull.* 18, no. 1 (2008): 8–10.

20. Amphetamines were a part of Paul Erdős' regimen: “10 to 20 milligrams of Benzedrine or Ritalin, strong espresso, and caffeine tablets.” See Paul Hoffman, *The Man Who Loved Only Numbers: The Story of Paul Erdős and the Search for Mathematical Truth* (New York: Hyperion, 1998), 7.

21. Littlewood and Bollobás, 200.

22. Jean-Pierre Serre, “How to Write Mathematics Badly,” https://www.youtube.com/watch?v=ECQyFzzBHlo. https://www.youtube.com/watch?v=ECQyFzzBHlo.

23. Dan Kalman, *Uncommon Mathematical Excursions: Polynomia and Related Realms* (Washington D.C.: MAA, 2009), 78.

24. “Many (I think most) papers in most refereed journals are not refereed. There is a presumptive referee who looks at the paper, reads the introduction and the statements of the results, glances at the proofs, and, if everything seems okay, recommends publication. Some referees do check proofs line-by-line, but many do not. When I read a journal article, I often find mistakes. Whether I can fix them is irrelevant. The literature is unreliable,” Melvyn B. Nathanson, “Desperately seeking mathematical proof,” in *The Best Writing on Mathematics 2010*, ed. Mircea Pitici (Princeton, NJ: Princeton University Press, 2011.)

25. Gian-Carlo Rota's graduate student recollections (Rota, 8–9) include the following origin story for this phenomenon:

> [William] Feller took umbrage when someone interrupted his lecture by pointing out some glaring mistake. He became red in the face and raised his voice, often to full shouting range. It was reported that on occasion he had asked the objector to leave the classroom. The expression ‘proof by intimidation’ was coined after Feller's lectures (by Mark Kac). During a Feller lecture, the hearer was made to feel privy to some wondrous secret, one that often vanished by magic as he walked out of the classroom at the end of the period. Like many great teachers, Feller was a bit of a con man.

Monosyllabic

Here is a fact:

If x is real and the cube of x less six times the square of x plus five times x plus six times x less six is twice x less two, then x must be one or four.

The proof goes like this:

See, the first three terms on the left side split as the square of x less five times x all times x less one. And more, the last two terms on the left side split as six times x less one, while the right side splits as two times x less one. Thus, if x were to be one, we have nought plus nought is nought, which is true. So, x may be one.

Else x is not one, and x less one is not nought. So we can times the whole thing by one on top of x less one to yield: the square of x less five times x plus six is two. Drop two from each side, and the square of x less five times x plus four is nought. Now this splits as x less four times x less one. Since we said x less one is not nought, x less four must be. So x is one or four, as was to be shown.

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In Siobhan Roberts' biography *Genius at Play: The Curious Mind of John Horton Conway*, the distinguished mathematician claims that he delivered an entire number theory lecture according to the rules of the "One Bit Word Game," which is an autological title for the monosyllabic constraint.[10]

In place of *eleven*—the only polysyllabic coefficient in the equation—I've written "five plus six." This trivial alteration led to the factorization that is the basis of the solution here. Why am I so surprised by this? Mathematical proofs often emerge as a sequence of individually inconsequential transformations governed by external constraints. It could even be that this is typical.

In the 1948 article, "The Place of Mathematics in the Classification of the Sciences," Raymond Queneau argues that mathematics is both method and game, "in the most precise terms, what's known as a *jeu d'esprit*." He concludes the paper by signaling a proto-Oulipian correspondence: "We might say, giving Art its ambiguous sense, that Science oscillates from Art to Game and Art from Game to Science."[11]

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Screenplay

FADE IN:

EXT. RENAISSANCE MILAN, ITALY DAY

A town square. Church bells ring the noon hour. A crowd has gathered. CARDANO's protegé, the young LUDOVICO FERRARI stands at the center, with his ASSISTANT to the side.

FERRARI

Let he who calls himself Niccolò Fontana Tartaglia of Brescia come forth and defend his honor. He who prints libelous attacks upon our esteemed professor Girolamo Cardano in a vain effort to bring disrepute to our University of Milan, the greatest in all Lombardy.

FERRARI looks over the crowd. PAN OVER as the crowd searches itself. PUSH IN to...

TOWNSPERSON

Brixians are chickens!

The crowd cheers. Finally, FOLLOW a small man wearing sandals and modest linen robes as he makes his way through the crowd.

TARTAGLIA

Here, here I am.

TOWNSPERSON

Ta-ta-tar-ta-ta-taglia!

The crowd jeers and makes way.

FERRARI
(To TARTAGLIA)

Are you prepared?

TARTAGLIA

I did not traverse one hundred forty-three thousand six hundred and seventy nine braccio from Braccia, no, Brescia to Milan...

The crowd laughs. FERRARI gestures to quiet them.

FERRARI

Let him finish!

TARTAGLIA

To give counting lessons to a servant boy.

The crowd "ooh"s.

FERRARI

Indeed, certainly you did not. No more can one count feathers on a boar or trotters on a chicken.

The crowd laughs.

TARTAGLIA

What?!

FERRARI

Take no offence, 'twas only an observation of fact. For the Milanese pace is thrice the Brescian, and thus your one hundred forty-three thousand six hundred and seventy nine braccio is our forty-seven thousand eight hundred and ninety-three brabucco.

The crowd cheers.

TOWNSPERSON

How you like 'em countin' lessons.

(Laughter)

TARTAGLIA
(To FERRARI)

Where is your master?

FERRARI

If his Excellency Signor Hieronimo Cardano suffered every fool to challenge his authority in the art of algebra, he'd never have time for his patients. Surely if your genius overflows the bounds of our physician of Milan, you can best this lowly creature of his.

FERARRI bows. TARTAGLIA nods.

FERRARI
(To his ASSISTANT)

I hereby declare that on this the tenth of August in the year fifteen hundred and forty-eight of our Lord in our fair city of Milan, the visitor Niccolò Fontana Tartaglia of Brescia challenges Girolamo Cardano, represented here by Ludovico Ferrari Esquire, to a duel of the mind. The dishonored shall pay the winner two hundred scudi.

FERRARI jingles a leather pouch over head. The crowd cheers.

ASSISTANT
(Continuing)

Both parties have exchanged thirty problems. As visitor, Tartaglia, you may begin by presenting your solution to the first problem, which I shall now read.

He unfurls a scroll.

ASSISTANT
(Reading)

A bankrupt merchant begins for the repayment of half his debt in three years. The agreement is that each year he pays the same proportion of the remainder. He now...

TARTAGLIA

What? I've never seen this...

FERRARI

Is there a problem? If you wish to drop your challenge, simply say so.

TARTAGLIA

N-n-no! Continue.

ASSISTANT
(Reading)

He now wishes to know the amount of the initial payment he must make in order that after three years he will have repaid half his capital plus a debt fee equal three quarter his first payment.

CLOSE ON TARTAGLIA. He mops his brow with a handkerchief.

TARTAGLIA
(V.O.)

Suppose the debt were two hundred. The first years payment...

CLOSE ON FERRARI'S PURSE.

TARTAGLIA
(V.O.)

By the second year he will pay it less half its square.

Images of dancing coins float with his mental calculations.

TARTAGLIA
(V.O.)

And the third year he pays the first payment less its square plus a quarter its cube--all of which must come to one hundred plus three quarters of the first...

TOWNSPERSON
(Tapping his walking stick on the ground)

Ta-ta-ta-ta...

TARTAGLIA sweeps the TOWNSPERSON's walking stick out from under him with his foot. The TOWNSPERSON collapses, much to the amusement of the crowd. Without missing a beat, TARTAGLIA picks up the walking stick and continues his calculation, tracing figures in the dust with its end. TRACK as he writes "cube and 9 cose equal 6 square and 4". The crowd is silent. Then "cube equal 3 cose and 2."

TARTAGLIA'S MOTHER
(V.O. in Italian verse)

In el secondo de cotesti atti Quando che'l cubo restasse lui solo Tu osservarai quest'altri contratti...

TARTAGLIA stops writing.

TOWNSPERSON

He's stumped!

TARTAGLIA
(To the TOWNSPERSON)

Half. He repays half his debt in the first year.

TARTAGLIA
(To the TOWNSPERSON)

Half. He repays half his debt in the first year.

ASSISTANT

Correct.

The crowd gasps. FERRARI-CLOSE. Sweating.

FERRARI
(Grabbing the scroll from his ASSISTANT)

Let me see that.

GIROLAMO CARDANO, disguised in monastic robes, peers out from behind a fruit vendor's cart.

TARTAGLIA
(Shouting into the crowd)

Cardano, where are you?

CARDANO leaps back behind the cart, takes out a pen and scribbles on a scrap of paper that he gives to a BOY. Without being noticed, the BOY darts through the crowd and delivers the message to FERRARI. He places it in the scroll, which he returns to the ASSISTANT.

FERRARI

Next question!

ASSISTANT
(Reading the scrap of paper)

Four times a number exceeds by two the product of its square and cube. What is the number?

TARTAGLIA

A qui-qui-quintic?

The crowd laughs.

TARTAGLIA
(to FERARRI)

I know my limitations. Do you yours?

ASSISTANT

What is your answer?

TARTAGLIA

What is yours?

ASSISTANT

Your query is out of turn. Answer or admit ignorance.

TARTAGLIA is silent, but then begins again his calculations in the sand. The crowd steps back to make more room. CARDANO elbows the head of the BOY to get a better view. TARTAGLIA nears the fruit cart, but stops abruptly.

TARTAGLIA

I cannot extract the root.

ASSISTANT

You admit ignorance then?

TARTAGLIA turns his back to FERRARI, and slowly walks away. He hands the stick to the TOWNSPERSON.

TARTAGLIA

Of course.

TOWNSPERSON

The chicken is cooked!

The crowd laughs and cheers.
TARTAGLIA-CLOSE. Brow furrowed.

TARTAGLIA
(V.O.)

At least I was honest in my ingnorance. No one, least of all Ferrari, could solve the problem.

REVERSE to FERRARI watching him leave. With TARTAGLIA nearly out of sight, CARDANO steps forward to examine TARTAGLIA's writing in the dust.

CARDANO
(Reading out loud)

One thousand and seventeen parts in two thousand...

Pressed by the crowd, FERRARI backs into CARDANO and tramples the solution.

CARDANO
(To FERRARI)

You ignoramous!

TARTAGLIA-CLOSE. The crowd in the distance. He dons a small but victorious smirk.

FADE OUT:

THE END

The hero genius is a one of the most well worn narratives in the dramatic depiction of mathematics, closely accompanied by the mad genius. This characterization dates at least as far back as antiquity, when Archimedes, unable to contain his excitement at discovering the principle of displacement in his bath was supposed to have run through the streets of Syracuse in his birthday suit shouting "Eureka! Eureka!" With his discovery, he solved Hiero's problem, and such challenges are another feature of mathematical dramatizations that persist to today. Surprisingly, the mathematical duel between Tartaglia and Cardano is part of historical record, though the dialogue here is based on cinematic tropes of today.

According to a series of public letters between Tartaglia and Cardano's protegé Ludovico Ferrari, after repeated entreaties from the latter, Tartaglia divulged his solution to the reduced cubic, which he had committed to memory in the form of a poem. Cardano promised not to publish Tartaglia's solution. Some time later Cardano found evidence that the mathematician Scipione del Ferro, a professor at the University of Bologna knew the formula prior to and independent of Tartaglia, and Cardano satisfied himself that it wouldn't be breaking his promise to publish the solution since it wasn't really Tartaglia's in the first place. Tartaglia saw it differently. In a series of pamphlets published at his own expense, he waged a war against Cardano and this was the background for the mathematical showdown that took place at ten o'clock in the morning on August 10, 1548 at the Church of Zoccolante in Milan. By all accounts, Cardano didn't show up to the duel, but his "creature," as Tartaglia called him, did.

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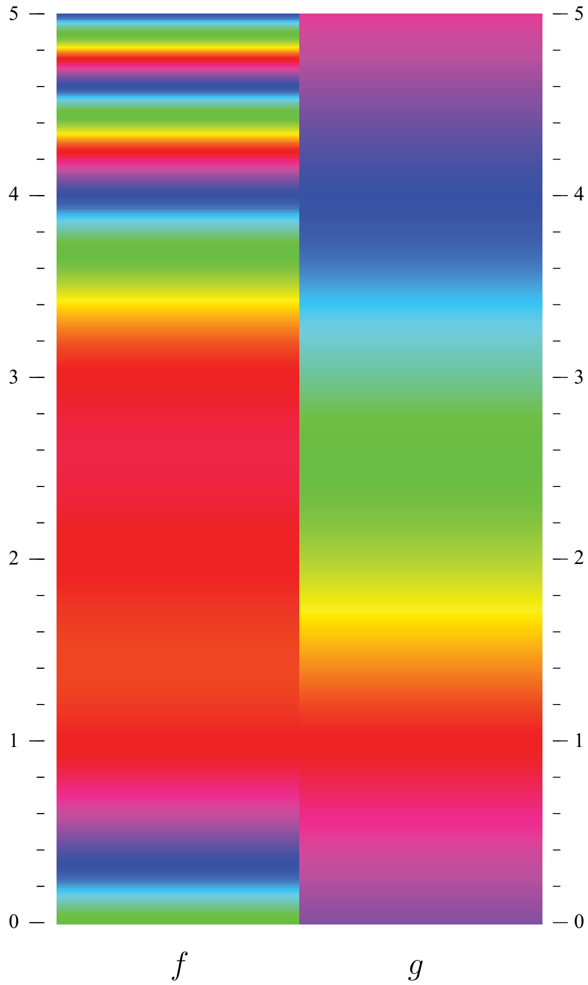
Chromatic



The two spectra represent the two sides of the equation

$$x^3 - 6x^2 + 11x - 6 = 2x - 2.$$

At height x the hue on the left is proportional to $f(x) = x^3 - 6x^2 + 11x - 6$ and the hue on the right is proportional to $g(x) = 2x - 2$. A unit corresponds to a difference of $40°$ in hue. The red band $(0°)$ at $x = 1$ and the blue band $(240°)$ at $x = 4$ are the two solutions.

This diagram is more typical of "false color" graphics appearing in the natural sciences, where color functions to accentuate and elucidate features of complicated data sets (think of a weather map). Some mathematicians also confront complicated data sets—especially since the advent of electronic computing—but journal articles are less likely to feature color graphics than one might expect.

The nineteenth century mathematician, engineer, and revolutionary Oliver Byrne thought this was an oversight and produced *The First Six Books of the Elements of Euclid in Which Coloured Diagrams and Symbols Are Used Instead of Letters for the Greater Ease of Learners* to prove it. His introduction makes a hard sell, claiming that students would absorb Euclid from his book in one third the time that it takes without color. He even defends against would-be detractors

> This Work has greater aim than mere illustration; we do not introduce colours for the purpose of entertainment, or to amuse *by certain combinations of tint and form*, but to assist the mind in its researches after truth, to increase the facilities of instruction, and to diffuse permanent knowledge.[12]

Some people can't help but see mathematics in color. Physicist Richard Feynman reported that "When I see equations, I see the letters in colors — I don't know why. As I'm talking, I see vague pictures of Bessel functions from Jahnke and Emde's book, with light-tan j's, slightly violet-bluish n's, and dark brown x's flying around. And I wonder what the hell it must look like to the students."[13]

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Arborescent

$$
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      \dfrac{
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            \dfrac{
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                \dfrac{}{6<x}
                \quad
                \dfrac{0<6 \quad \dfrac{}{6<x}}{0<x}
              }{6\cdot x < x\cdot x}
              \quad
              \dfrac{0<6 \quad \dfrac{}{6<x}}{0<x}
            }{6x\cdot x < x^2\cdot x}
          }{6x^2+2x < x^2\cdot x + 2x}
        }{6x^2+2x < x^3+2x}
      }{6x^2+2x+6 < x^3+2x+6}
      \quad
      \dfrac{}{6<x}
    }{6x^2+2x+6 < x^3+2x+x}
  }{6x^2+2x+6 < x^3+3x}
  \quad
  \dfrac{
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        0<2
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        \dfrac{
          8\cdot 0 = 0
          \quad
          \dfrac{
            0<8
            \quad
            \dfrac{0<6 \quad \dfrac{}{6<x}}{0<x}
          }{8\cdot 0 < 8\cdot x}
        }{0 < 8\cdot x}
      }{0 < 8\cdot x + 2}
    }{x^3+3x < x^3+3x+8x+2}
  }{x^3+3x < x^3+11x+2}
}{
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\dfrac{6x^2+2x+6 < x^3+11x+2}{2x+6 < x^3-6x^2+11x+2}
}{2x < x^3-6x^2+11x-6+2}
}{2x-2 < x^3-6x^2+11x-6}
}{2x-2 \neq x^3-6x^2+11x-6}
}{x^3-6x^2+11x-6 \neq 2x-2}
}
$$

Direct computation for $x = 0, 2, 3, 5, 6$ yields the same conclusion.
Hence $x \in \mathbb{N}$, $x^3 - 6x^2 + 11x - 6 = 2x - 2$ implies $x = 1$ or $x = 4$.

This proof is meant to be read from top to bottom, from "leaf" to "trunk." Each horizontal line represents a deduction with the statement beneath following from the premise(s) above. If there are two premises over the same line they are conjoined by an invisible "and." The empty premise above the inequality $6 < x$ indicates that it is the assumption of the proof. The expression $x \in \mathbb{N}$ in the final line is the hypothesis that x is an element of the set of natural numbers. This variation is modeled on the *Gentzen-style* proof calculus of natural deduction, named for the German logician, Gerhard Gentzen.[14]

Paranoid

a a a a a a a a a a a a a a a a a a a a a a a a a a a a a a a a a a a b b b b b b b c c c c c c c c c c c c c d d d d d d d d d d d d d d d e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e e f f f f f f f f f f f f g g g g g g g g g h h h h h h h h h h h h h h i i i i i i i i i i i i i i i i i i i i i i i i i i i k k l l l l l l l l l l l l l l l m m m m m m m m m m m m m m n n n n n n n n n n n n n n n n n n n n n n n n n n o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o p p p p p p p p p q q q q q q r r r r r r r r r r r r r r r r r r r r r r r r r s s s s s s s s s s s s s s s s s s s s s s s s s s s s s s s s s s s t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t t u u u u u u u u u u u u u u u u u u u u u v v w w w w w w x x x x x x x y y y y y y

This proof contains all the letters used by another variation in this book arranged in alphabetical order.

Galileo used anagrams as a way to ensure priority of a discovery at the outset thus affording him time to verify his results before claiming them more publicly. For example, observing Saturn in 1610, he sent Kepler the jumble of letters

$$smaismrmilmepoetaleumibunenugttauiras$$

that could be rearranged into the Latin for "I observed the highest planet to be triple-bodied."[15] With more powerful telescopes, the extra bodies about Saturn were revealed to be its rings. The Dutch astronomer and mathematician Christiaan Huygens made this discovery in 1656 and published it in the form of an anagram also, this time arranging the letters in alphabetic order:

$$aaaaaaacccccdeeeeehiiiiiiillllmmnnnnnnnnnooooppqrrstttttuuuuu.$$ [16]

A famously secretive mathematician, Isaac Newton encoded his discovery of the calculus when describing one of its applications in a 1676 letter to the German theologian and natural philosopher Henry Oldenburg.[17] In his shorthand, coefficients count the more frequently appearing letters,

> At present I have thought fit to register [my methods] by transposed letters, lest through others obtaining the same result, I should be compelled to change the plan in some respects.

$$5accd\text{æ}10effh11i4l3m9n6oqqr8s11t9y3x:$$
$$11ab3cdd10e\text{æ}g10ill4m7n6o3p3q6r5s11t8vx,$$
$$3ac\text{æ}4egh5i4l4m5n8oq4r3s6t4v,$$
$$aadd\text{æ}ecceeeiijmmnnooprrrsssssttuu.$$

This little history of competition and paranoia left me wondering if these weren't unavoidable features of modern mathematical practice. While this could be true, they may just signal periods of mathematical growth, modern or not. Apparently Archimedes planted false theorems in his mathematical letters so as to defend himself against lesser-minded plagiarists.[18]

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Psychedelic

The image was created from Newton's method for finding roots. Newton's method applies a function to an initial estimate of a root and returns another, hopefully improved, root approximation. By applying the function to this second estimate, and so on, one obtains a sequence of approximations that often converge to an actual root. The image in this proof was obtained by taking points of the *complex* plane as initial estimates. After some number of iterations, the method agrees with one or the other root to within four decimal places. The wavey "C" curve on the right side of the image separates initial points converging to 1 (left) from those points converging to 4 (right). If the number of iterations needed to arrive (sufficiently close) to a root is even/odd then the point is colored black/white.

The inspiration for this style came from an article entitled "Mathematics and the Psychedelic Revolution" by Ralph Abraham of the Department of Mathematics at UC Santa Cruz.[19] The fascinating account appeared in the *Bulletin of the Multidisciplinary Association for Psychedelic Studies*:

> It all began in 1967 when I was a professor of mathematics at Princeton, and one of my students turned me on to LSD. That led to my moving to California a year later, and meeting at UC Santa Cruz a chemistry graduate student who was doing his Ph.D. thesis on the synthesis of DMT. He and I smoked up a large bottle of DMT in 1969, and that resulted in a kind of secret resolve, which swerved my career toward a search for the connections between mathematics and the experience of the logos, or what Terence calls "the transcendent other." This is a hyperdimensional space full of meaning and wisdom and beauty, which feels more real than ordinary reality, and to which we have returned many times over the years, for instruction and pleasure. In the course of the next twenty years there were various steps I took to explore the connection between mathematics and the logos…There is no doubt that the psychedelic revolution in the 1960s had a profound effect on the history of computers and computer graphics, and of mathematics, especially the birth of post-modern maths such as chaos theory and fractal geometry. This I witnessed personally. The effect on my own history, viewed now in four decades of retrospect, was a catastrophic shift from abstract pure math to a more experimental and applied study of vibrations and forms, which continues to this day.

The use of prescription-strength stimulants among mathematicians is probably somewhat more common (though how common would be difficult to know) than the use of hallucinogenics.[20] J. E. Littlewood offered a more cautiously optimistic note on academic drug use: "I can envisage a future in which stimulant drugs could raise mental activity for a set period of work, and relaxing ones give a suitable compensating period, perhaps of actual sleep. The present is a time of transition; stimulants do exist; but they should be used only with the greatest care and only in a crisis. And there is the problem of knowing what *is* a crisis."[21]

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Authority

Of course, if $x^3 - 6x^2 + 11x - 6 = 2x - 2$, then it follows from Euler that the real number in question must be 1 or 4.

The eighteenth century Swiss mathematician and physicist Leonhard Euler was perhaps the most prolific mathematician of all time. One of Jean-Pierre Serre's tips from a lecture on how to write mathematics badly is that "when you give a reference, a reference that you don't want to be checked…refer to the entire works of Euler. They have not yet been published entirely."[22]

While Euler did contribute a solution to the general cubic,[23] appeals to authority don't require that the authority has actually proven the result. It may not even require that the result is true. In an opinion piece in the *Notices of the AMS*, American mathematician Melvyn Nathanson makes the following speculation:

> How do we recognize mathematical truth? If a theorem has a short complete proof, we can check it. But if the proof is deep, difficult, and already fills 100 journal pages, if no one has the time and energy to fill in the details, if a "complete" proof would be 100,000 pages long, then we rely on the judgments of the bosses in the field. In mathematics, a theorem is true, or it's not a theorem. But even in mathematics, truth can be political.[24]

When the authority of reference is oneself, a proof by authority becomes a proof by *intimidation*.[25]

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